Code - ❍

ANSWERS & HINT
for
WBJEE - 2018
SUB : MATHEMATICS

CATEGORY - I (Q1 to Q50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ marks will be deducted.

1. The approximate value of $\sin 31^\circ$ is

(A) > 0.5 (B) > 0.6 (C) < 0.5 (D) < 0.4

Ans : (A)

Hint : $\sin 30^\circ < \sin 31^\circ < \sin 37^\circ$

$\Rightarrow 0.5 < \sin 31^\circ < 0.6$

2. Let $f_1(s) = e^x$, $f_2(x) = e^{f_1(x)}, \ldots\ldots\ldots f_{n+1}(x) = e^{f_n(x)}$ for all $n \geq 1$. The for any fixed n, $\frac{d}{dx}f_n(x)$ is

(A) $f_n(x)$ (B) $f_n(x)\,f_{n-1}(x)$ (C) $f_n(x)\,f_{n-1}(x) \ldots\ldots f_1(x)$ (D) $f_n(x) \ldots\ldots\ldots\ldots f_1(x)e^x$

Ans : (C)

Hint : $\frac{d}{dx}f_n(x) = \frac{d}{df_{n-1}(x)}e^{f_{n-1}(x)} \times \frac{d}{dx}f_{n-1}(x)$

$= f_n(x) \times \frac{d}{df_{n-2}(x)}e^{f_{n-2}(x)} \times \frac{d}{dx}f_{n-2}(x)$

$= f_n(x)f_{n-1}(x)f_{n-2}(x) \times \ldots\ldots\ldots\ldots \times f_2(x) \times \frac{d}{dx}f_1(x)$

$= f_n(x)f_{n-1}(x)f_{n-2}(x)\ldots\ldots\ldots\ldots f_1(x)$

3. The domain of definition of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$ is

(A) $(-\infty, -1) \cup (2, \infty)$ (B) $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$

(C) $(-\infty, 1) \cup (2, \infty)$ (D) $[-1, 1] \cup (2, \infty)$

Here $(a, b) \equiv \{x : a < x < b\}$ & $[a, b] \equiv \{x : a \leq x \leq b\}$

Ans : (B)

Hint : $\frac{1-|x|}{2-|x|} \geq 0 \Rightarrow \frac{|x|-1}{|x|-2} \geq 0 \Rightarrow |x| \leq 1$ as $|x| > 2$

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty) \cup [-1, 1]$

4. Let $f : [a, b] \to \mathbb{R}$ be differentiable on [a, b] & $k \in \mathbb{R}$. Let f(a) = 0 = f(b)
Also let $J(x) = f'(x) + k f(x)$. Then
(A) $J(x) > 0$ for all $x \in [a, b]$
(B) $J(x) < 0$ for all $x \in [a, b]$
(C) $J(x) = 0$ has at least one root in (a, b)
(D) $J(x) = 0$ through (a, b)

Ans : (C)

Hint : Let g(x) = kx f(x) which is continuous in [a, b] and differentiable in (a, b) g(a) = 0 = g(b))
$\Rightarrow g'(c) = 0$ for same $c \in (a, b)$ (by Rolle's theorem)
$\Rightarrow kf(c) + kcf'(c) = 0$
$\Rightarrow j(x) = 0$ for same $x = c \in (a, b)$

5. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$

(A) does not exsit
(B) is $\frac{50}{3}$
(C) is $\frac{53}{3}$
(D) is $\frac{22}{3}$

Ans : (C)

Hint : $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h} = \lim_{h\to 0}\frac{f(1)-f(1-h)}{-h} \times \frac{h}{h^3+3h}$

$= -f'(1) \times \frac{1}{3} = \frac{53}{3}$

6. Let $f : [a, b] \to \mathbb{R}$ be such f is differentiable in (a, b), f is continuous at x = a & x = b and moreover f(a) = 0 = f(b). Then
(A) there exists at least one point c in (a, b) such that $f'(c) = f(c)$
(B) $f'(x) = f(x)$ does not hold at any poit in (a, b)
(C) at every point of (a, b), $f'(x) > f(x)$
(D) at every point of (a, b), $f'(x) < f(x)$

Ans : (A)

Hint : Let, $g(x) = e^{-x} f(x)$ which is continuous in [a, b] and differentiable in (a, b)
Now, g(a) = g(b) = 0
$\Rightarrow \exists\, k \in (a, b)$ so that
$g'(k) = 0 \Rightarrow e^{-k}f'(k) - e^{-k}f(k) = 0$
$\Rightarrow f'(k) = f(k)$

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$. Then
(A) $f''(0) = 0$
(B) $f''(c) = 0$ for some $c \in \mathbb{R}$
(C) if $c \neq 0$, then $f''(c) \neq 0$
(D) $f'(x) > 0$ for all $x \neq 0$

Ans : (B)

Hint : f(0) = f(1) = 0
$\Rightarrow f'(k) = 0$ for same $k \in (0, 1)$ by Rolle's Theorem.
Again, $f'(o) = f'(k) = 0$
$\Rightarrow f''(c) = 0$ for same $c \in (0, k)$ by Rolle's Theorem.

8. If $\int e^{\sin x}\cdot\left[\frac{x\cos^3 x - \sin x}{\cos^2 x}\right]dx = e^{\sin x}.f(x) + c$, where c is constant of integration, then f(x) =

(A) sec x – x (B) x – sec x (C) tan x – x (D) x – tan x

Ans : (B)

Hint : $e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right) = e^{\sin x}\left(x\cos x - \sec x\tan x\right)$

$= e^{\sin x}\left(x\cos x - 1 + 1 - \sec x\tan x\right) = e^{\sin x}\times\cos x(x - \sec x) + e^{\sin x}(1 - \sec x\tan x)$

$= d[e^{\sin x}(x - \sec x)]$

9. If $\int f(x)\ \sin x\cos x\, dx = \frac{1}{2(b^2 - a^2)}\log f(x) + c$, where c is the constant of integration, then f(x) =

(A) $\frac{2}{(b^2 - a^2)\sin 2x}$ (B) $\frac{2}{ab\sin 2x}$ (C) $\frac{2}{(b^2 - a^2)\cos 2x}$ (D) $\frac{2}{ab\cos 2x}$

Ans : (C)

Hint : $\frac{d}{dx}\left[\frac{1}{2(b^2 - a^2)}\log f(x) + c\right]$

$= \frac{f'(x)}{f(x)}\times\frac{1}{2(b^2 - a^2)} = f(x)\sin x\cos x$ (by question)

$\Rightarrow y^2\sin 2x = \frac{dy}{dx}\ \frac{1}{b^2 - a^2}$ (Take f(x) = y)

$\Rightarrow \int\frac{dy}{y^2} = (b^2 - a^2)\int\sin 2x\, dx \Rightarrow y = \frac{2}{(b^2 - a^2)\cos 2x}$

10. If $M = \int_0^{\pi/2}\frac{\cos x}{x + 2}dx$, $N = \int_0^{\pi/4}\frac{\sin x\cos x}{(x + 1)^2}dx$, then the value of M – N is

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{2}{\pi - 4}$ (D) $\frac{2}{\pi + 4}$

Ans : (D)

Hint : $N = \int_0^{\pi/4}\frac{\sin 2x dx}{2(x + 1)^2}$

let $2x = t$, $dt = 2dx$, $N = \int_0^{\pi/2}\frac{\sin t\frac{dt}{2}}{2\frac{(t + 2)^2}{4}} \Rightarrow N = \int_0^{\pi/2}\frac{\sin t\, dt}{(t + 2)^2} = \left(\frac{-\sin t}{t + 2}\right)_0^{\pi/2} + \int_0^{\pi/2}\frac{\cos t}{(t + 2)}dt$

$\Rightarrow N = \frac{-2}{\pi + 4} + M \Rightarrow M - N = \frac{2}{\pi + 4}$

11. The value of the integral $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$ is

(A) $\frac{\pi}{4}\log 2014$ (B) $\frac{\pi}{2}\log 2014$ (C) $\pi \log 2014$ (D) $\frac{1}{2}\log 2014$

Ans : (B)

Hint : $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx = \int_{2014}^{1/2014} \tan^{-1} 1/t \times t \times \left(-\frac{1}{t^2}\right) dt$ (put $x = 1/t$)

$= \int_{1/2014}^{2014} \frac{\cot^{-1} t}{t} dt$

$\Rightarrow 2I = \pi/2 \int_{1/2014}^{2014} \frac{dx}{x} \Rightarrow I = \frac{\pi}{4} \times 2\log 2014 = \frac{\pi}{2}\log 2014$

12. Let $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$. Then

(A) $\frac{1}{2} \le I \le 1$ (B) $4 \le I \le 2\sqrt{30}$ (C) $\frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$ (D) $1 \le I \le \frac{2\sqrt{3}}{\sqrt{2}}$

Ans : (C)

Hint : $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$

$\Rightarrow \frac{3}{\pi} \times \sin\frac{\pi}{3} \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \le I \le \frac{4}{\pi} \times \sin\frac{\pi}{4} \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\Rightarrow \frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$

13. The value of $I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$, is

(A) 1 (B) π (C) e (D) $\pi/2$

Ans : (B)

Hint : $I = \int_0^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\cos x)} + e^{\tan^{-1}(\sin x)}} dx \quad - \int_0^{\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \Rightarrow I = \frac{5\pi}{4} - \frac{\pi}{4} \Rightarrow I = \pi$

14. The value of $\lim_{n\to\infty} \frac{1}{n}\left\{\sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \dots\dots + \sec^2\frac{n\pi}{4n}\right\}$ is

(A) $\log_e 2$ (B) $\frac{\pi}{2}$ (C) $\frac{4}{\pi}$ (D) e

Ans : (C)

Hint : Required limit $= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\sec^2\left(\frac{r\pi}{4n}\right) = \int_0^1 \sec^2\left(\frac{\pi x}{4}\right) dx = \frac{4}{\pi}\tan\left(\frac{\pi x}{4}\right)\Big|_0^1 = \frac{4}{\pi}$

15. The differential equation representing the family of curves $y^2 = 2d\left(x+\sqrt{d}\right)$ where d is a parameter, is of

(A) order 2 (B) degree 2 (C) degree 3 (D) degree 4

Ans : (C)

Hint : $y^2 = 2dx + 2d^{3/2}$

$$2y\frac{dy}{dx} = 2d$$

Degree $= 3$

16. Let y(x) be a solution of $\left(1+x^2\right)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and y(0) = –1. Then y(1) is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) –1

Ans : (C)

Hint : $\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$, $IF = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln\left(1+x^2\right)} = 1+x^2$ $\therefore$ $y = \frac{4x^3}{3\left(1+x^2\right)} - \frac{1}{1+x^2}$ $\therefore$ $y(1) = \frac{4}{6} - \frac{1}{2} = \frac{2}{3} - \frac{1}{2}$

$= \frac{1}{6}$

17. The law of motion of a body moving along a straight line is $x = \frac{1}{2}vt$, x being its distance from a fixed point on the line at time t and v is its velocity there. Then

(A) acceleration f varies directly with x
(B) acceleration f varies inversely with x
(C) acceleration f is constant
(D) acceleration f varies directly with t

Ans : (A)

Hint : $x = \frac{1}{2}vt \Rightarrow \frac{dx}{dt} = \frac{1}{2}\left[v + t\frac{dv}{dt}\right] \Rightarrow v = \frac{v}{2} + \frac{1}{2}t\,f \Rightarrow f = \frac{v}{t} \Rightarrow f = \frac{2x}{t^2}$

18. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (B)

Hint : $y = x^2$

$P(\alpha,\alpha^2)$ is a point on this parabola .

$\therefore y - \alpha^2 = 2\alpha(x-\alpha)$

$y = 2\alpha x - \alpha^2$ -----(1) is a tangent

$\therefore 2\alpha x - \alpha^2 = -x^2 + 4x - 4$

$x^2 + 2x(\alpha-2) + (4-\alpha^2) = 0$

Discriminant = 0

$4(\alpha-2)^2 - 4(4-\alpha^2) = 0$

$\alpha^2 - 4\alpha + \cancel{4} - \cancel{4} + \alpha^2 = 0$

$\alpha^2 - 2\alpha = 0$

$\alpha = 0,\ \alpha = 2$

19. Given that n number of A.Ms are inserted between two sets of numbers a, 2b and 2a, b where a, b $\in \mathbb{R}$. Suppose further that the m^{th} means between these sets of numbers are same, then the ratio a : b equals

(A) n – m + 1 : m (B) n – m + 1 : n (C) n : n – m + 1 (D) m : n – m + 1

Ans : (D)

Hint : 2b = (n+2)th term = a+(n+1)d

$$d = \frac{2b-a}{n+1}$$

$\therefore$ mth mean = $a + m\left(\frac{2b-a}{n+1}\right)$

Similarly b = (n+2)th term = 2a+(n+1)d

$$d = \frac{b-2a}{n+1}$$

$\therefore$ mth mean = 2a+m $\left(\frac{b-2a}{n+1}\right)$

$$\therefore\ a + m\left(\frac{2b-a}{n+1}\right) = 2a + m\left(\frac{b-2a}{n+1}\right)$$

$$\frac{a}{b} = \frac{m}{n+1-m}$$

20. If x + $\log_{10}(1+2^x)$ = x $\log_{10}5$ + $\log_{10}6$ then the value of x is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 1 (D) 2

Ans : (C)

Hint : $x + \log_{10}\left(1+2^x\right) = x\log_{10}5 + \log_{10}6$

$$\log_{10}\left(10^x\right) + \log_{10}\left(1+2^x\right) = \log_{10}5^x + \log_{10}6$$

$2^x(1+2^x) = 6$

$y(1+y) = 6 \Rightarrow y = 2 \Rightarrow 2^x = 2 \Rightarrow x = 1$

21. If $Z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2\pi r}{11}$ then $\sum_{r=0}^{10} Z_r =$

(A) –1 (B) 0 (C) i (D) –i

Ans : (B)

Hint : $Z_r = -i\left(\cos\frac{2\pi r}{11} + i\sin\frac{2\pi r}{11}\right) = -i\ e^{i\frac{2\pi r}{11}}$

$$\Rightarrow \sum_{r=0}^{10} Z_r = -i\sum_{r=0}^{10} e^{i\frac{2\pi r}{11}} = -i\times 0 = 0$$

22. If z_1 and z_2 be two non zero complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$, then the origin and the points represented by z_1 and z_2
(A) lie on a straight line
(B) form a right angled triangle
(C) form an equilateral triangle
(D) from an isosceles triangle

Ans : (C)

Hint : $z_1^2 + z_2^2 + z_2^3 = z_1z_2 + z_2z_3 + z_3 z_1$ is the condition for $z_1z_2z_3$ to be the vertices of an equilateral triangle.
Putting $z_3 = 0$
$z_1^2 + z_2^2 = z_1z_2$

23. If $b_1b_2 = 2(c_1+c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations $x^2+b_1x + c_1 = 0$ and $x^2+b_2x+c_2 = 0$ has
(A) real roots
(B) purely imaginary roots
(C) roots of the form a+ib $(a, b \in \mathbb{R}, ab \neq 0)$
(D) rational roots

Ans : (A)

Hint : $D_1 = b_1^2 - 4c_1$
$D_2 = b_2^2 - 4c_2$

$D_1+D_2 = b_1^2 + b_2^2 - 4(c_1+c_2) \quad = b_1^2 + b_2^2 - 2b_1b_2 = (b_1-b_2)^2 \geq 0$

$\therefore$ At least one of D_1, D_2 non-negative.

24. The number of selection of n objects from 2n objects of which n are identical and the rest are different is
(A) 2^n
(B) 2^{n-1}
(C) 2^n-1
(D) $2^{n-1}+1$

Ans : (A)

Hint : Ways of selections are
n identical and no different = 1 way
n–1 identical and one from different elements $= 1\times n_{c_1}$
.
.
.
.
0 identical rest from different $= 1\times {}^nc_n$

$\sum = {}^nc_0 + {}^nc_1 + {}^nc_2 + ... + {}^nc_n \quad = 2^n$

25. If $(2 \leq r \leq n)$, then ${}^nC_r + 2.\ {}^nC_{r+1} + {}^nC_{r+2}$ is equal to
(A) $2.\ {}^nC_{r+2}$
(B) ${}^{n+1}C_{r+1}$
(C) ${}^{n+2}C_{r+2}$
(D) ${}^{n+1}C_r$

Ans : (C)

Hint : ${}^nC_r + 2\ {}^nC_{r+1} + {}^nC_{r+2}$
$= {}^nC_r + {}^nC_{r+1} + {}^nC_{r+1} + {}^nC_{r+2} = {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2} = {}^{n+2}C_{r+2}$

26. The number $(101)^{100} - 1$ is divisible by
(A) 10^4
(B) 10^6
(C) 10^8
(D) 10^{12}

Ans : (A)

Hint : $(101)^{100} - 1 = (1 + 100)^{100} - 1$
$= [\ 1 + {}^{100}C_1\ 100 + {}^{100}C_2\ 100^2 + ...+{}^{100}C_{100}(100)^{100}]-1 = 10^4\ (1 + {}^{100}C_2 + {}^{100}C_3 10^2 + ...+{}^{100}C_{100}(100)^{98})$
$= 10^4$ (1 + an integer multiple of 10)

27. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may also have the greatest coefficient is

(A) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (B) $\frac{n}{n+1} < x < \frac{n+1}{n}$ (C) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$ (D) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

Ans : (A)

Hint : $\frac{n}{2} < \frac{x(n+1)}{x+1} < \frac{n}{2}+1 \Rightarrow \frac{n}{n+2} < x < \frac{n+2}{n}$

28. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$ = A, then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

(A) A^2 (B) $A^2 - A + I_3$ (C) $A^2 - 3A + I_3$ (D) $3A^2 + 5A - 4I_3$

Ans : (A)

Hint : $P = \begin{bmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{bmatrix}$

then adj P = $\begin{bmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{bmatrix}$

$A = |P|$ and $A^2 = |\text{Adj}\ (P)|^2$

29. If $a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(A) 1 (B) –1 (C) 0 (D) 2

Ans : (C)

Hint : Let $\alpha = e^{i2\pi/9}$ then given determinant = $\begin{vmatrix} \alpha & \alpha^2 & \alpha^3 \\ \alpha^4 & \alpha^5 & \alpha^6 \\ \alpha^7 & \alpha^8 & \alpha^9 \end{vmatrix} = \alpha^{12} \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha & \alpha^2 \end{vmatrix} = 0$

30. If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of

(A) x only (B) y only (C) n only (D) x, y, z and n

Ans : (D)

Hint : $\sum_{r=1}^{n} S_r = \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix} = 0$

31. If the following three linear equations have a non-trivial solution, then
$x + 4ay + az = 0$
$x + 3by + bz = 0$
$x + 2cy + cz = 0$
(A) a, b, c are in A.P. (B) a, b, c are in G.P (C) a, b, c are in H.P. (D) $a + b + c = 0$

Ans : (C)

Hint : To have non-trivial solution, $\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0 \Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

32. On $\mathbb{R}$, a relation ρ is defined by $x\rho y$ if and only if $x - y$ is zero or irrational. Then
(A) ρ is equivalence relation (B) ρ is reflexive but neither symmetric nor transitive
(C) ρ is reflexive and symmetric but not transitive (D) ρ is symmetric and transitive but not reflexive

Ans : (C)

Hint : On the set $\mathbb{R}$

$x\rho y \Leftrightarrow x - y = 0$ or $x - y \in Q^c \because x - x = 0 \Rightarrow x\rho x$ (Reflexive)

if $x - y = 0 \Rightarrow y - x = 0$ or $x - y \in Q^c \Rightarrow y - x \in Q^c$ (Symmetric)

Take $x = 1 + \sqrt{2}$; $y = \sqrt{2} + \sqrt{3}$; $z = \sqrt{2} + 2$

$x - y = 1 - \sqrt{3} \in Q^c$ and $y - z = \sqrt{3} - 2 \in Q^c$

Here xRy and yRz but x is not related to z $\therefore$ Not transitive

33. On the set $\mathbb{R}$ of real numbers, the relation ρ is defined by $x\rho y$, $(x, y) \in \mathbb{R}$
(A) If $|x-y| < 2$ then ρ is reflexive but neither symmetric nor transitive
(B) If $x - y < 2$ then ρ is reflexive and symmetric but not transitive
(C) If $|x| \geq y$ then ρ is reflexive and transitive but not symmetric
(D) If $x > |y|$ then ρ is transitive but neither reflexive nor symmetric

Ans : (D)

Hint : for option D, $x > |x|$ is not true hence not reflexive

Take $x = 2$, $y = -1$, clearly $x > |y|$ but $y > |x|$ doe not hold hence not symmetric

Now, Let $x > |y|$ and $y > |z| \Rightarrow x, y > 0$. $\therefore$ Rewriting, $x > |y|$ and $y > |z| \Rightarrow x > |z|$ hence transitive

34. If $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^x$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^2$. The mapping $g \circ f : \mathbb{R} \to \mathbb{R}$ be defined by $(g \circ f)(x) = g[f(x)]\ \forall\ x \in \mathbb{R}$, Then
(A) $g \circ f$ is bijective but f is not injective (B) $g \circ f$ is injective and g is injective
(C) $g \circ f$ is injective but g is not bijective (D) $g \circ f$ is surjective and g is surjective

Ans : (C)

Hint : g is neither injective nor surjective

$g \circ f(x) = e^{2x}$, $x \in \mathbb{R} \therefore g \circ f$ is injective

35. In order to get a head at least once probability ≥ 0.9, the minimum number of time a unbiased coin needs to be tossed is
(A) 3 (B) 4 (C) 5 (D) 6

Ans : (B)

Hint : Let x = no. of heads appear in n tossed

$X \sim \text{Bin}\left(n, \frac{1}{2}\right)$

Now, $P(x \geq 1) = 1 - P(x = 0) = 1 - \frac{1}{2^n} \geq 0.9 \Rightarrow \frac{1}{2^n} \leq \frac{1}{10} \Rightarrow n \geq 4$

$\therefore$ minimum number of tosses = 4

36. A student appears for tests I, II and III. The student is successful if he passes in tests I, II or I, III. The probabilities of the student passing in tests, I, II and III are respectively p, q and $\frac{1}{2}$. If the probability of the student to be successful is $\frac{1}{2}$. Then

(A) p (1 + q) = 1 (B) q (1+p) = 1 (C) pq = 1 (D) $\frac{1}{p}+\frac{1}{q}=1$

Ans : (A)

Hint : Let x_1 - He passes in test-I

x_2 - He passes in test-II

x_3 - He passes in best - III

x - He is successful

$$x \equiv (x_1 \cap x_2 \cap x_3') \cup (x_1 \cap x_2' \cap x_3) \cup (x_1 \cap x_2 \cap x_3)$$

$$\therefore\ p(x) = p(x_1).p(x_2).p(x_3') + p(x_1).p(x_2').p(x_3) + p(x_1).p(x_2).p(x_3)$$

$$\Rightarrow \frac{1}{2} = pq.\frac{1}{2} + p\,(1-q)\frac{1}{2} + pq\frac{1}{2} \Rightarrow p + pq = 1 \therefore p(1+q)=1$$

37. If sin 6θ + sin 4θ + sin 2θ = 0, then general value of θ is

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$ (C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (n is integer) (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Ans : (A)

Hint : sin 6θ + sin 4θ + sin 2 = θ0 ⇒ 2 sin 4θ. cos 2θ + sin 4 θ = 0 ⇒ sin 4 θ = 0 or 2 cos 2θ + 1 = 0

$\Rightarrow 4\theta = n\pi$ $\quad \Rightarrow \cos 2\theta = -\frac{1}{2}$

$\Rightarrow \theta = \frac{n\pi}{4}, (n \in \mathbb{Z})$ $\quad \Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3}$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3} (n \in \mathbb{Z})$$

38. If $0 \le A \le \frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ is equal to

(A) $\frac{\pi}{4}$ (B) π (C) 0 (D) $\frac{\pi}{2}$

Ans : (B)

39. Without changing the direction of the axes, the origin is transferred to the point (2, 3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

(A) $x^2 + y^2 + 4 = 0$ (B) $x^2 + y^2 = 4$

(C) $x^2 + y^2 - 8x - 12y + 48 = 0$ (D) $x^2 + y^2 = 9$

Ans : (B)

Hint : $x \to x + 2$, $y \to y + 3$ in given curve

$\Rightarrow x^2 + y^2 = 4$

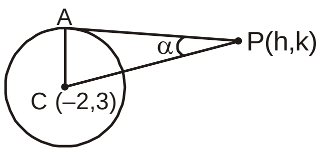
40. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is

(A) $x^2 + y^2 + 4x + 6y + 9 = 0$ (B) $x^2 + y^2 - 4x + 6y + 9 = 0$

(C) $x^2 + y^2 - 4x - 6y + 9 = 0$ (D) $x^2 + y^2 + 4x - 6y + 9 = 0$

Ans : (D)

Hint :



$\because \sin\alpha = \dfrac{CA}{CP}$

$\Rightarrow [(h+2)^2 + (k-3)^2]\sin^2\alpha = 4\sin^2\alpha$

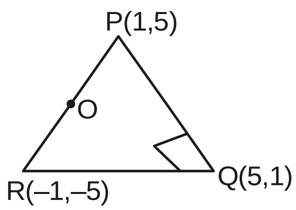
$\Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$

41. The point Q is the image of the point P(1, 5) about the line y = x and R is the image of the point Q about the line y = –x. The circumcenter of the ΔPQR is

(A) (5, 1) (B) (–5, 1) (C) (1, –5) (D) (0, 0)

Ans : (D)

Hint :



$\because \angle Q = 90^\circ$ $\Rightarrow$ Circumcentre = mid point of P and R i.e. (0, 0)

42. The angular points of a triangle are A (–1, –7), B(5, 1) and C(1, 4). The equation of the bisector of the $\angle ABC$ is

(A) x = 7y + 2 (B) 7y = x + 2 (C) y = 7x + 2 (D) 7x = y + 2

Ans : (B)

Hint : Equation of AB : 3y – 4x + 17 = 0

Equation of BC : 4y + 3x – 19 = 0

Equation of bisectors

4y + 3x – 19 = ± (3y – 4x + 17) (using position of points A and C with respect to bisectors)

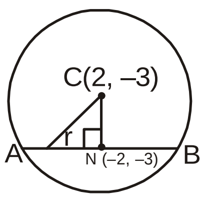
7y = x + 2

43. If one of the diameters of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2, –3), the radius of S is

(A) $\sqrt{41}$ unit (B) $3\sqrt{5}$ unit (C) $5\sqrt{2}$ unit (D) $2\sqrt{5}$ unit

Ans : (A)

Hint :



Equation chord AB : x = –2

$\therefore$ A (–2 + r cos90°, –3 + r sin 90°) $\equiv$ A(–2, r – 3)

Putting A(–2, r – 3) in the circle

$\Rightarrow r = \pm$ $\Rightarrow |r| = 5$

CN = 4 $\therefore CA = \sqrt{41}$

44. A chord AB is drawn from the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, and is extended to M such that AM = 2AB. The locus of M is

(A) $x^2 + y^2 - 8x - 6y + 9 = 0$ (B) $x^2 + y^2 + 8x + 6y + 9 = 0$

(C) $x^2 + y^2 + 8x - 6y + 9 = 0$ (D) $x^2 + y^2 - 8x + 6y + 9 = 0$

Ans : (C)

Hint :

$\because$ AM = 2AB $\Rightarrow B\left(\frac{x}{2}, \frac{y+3}{2}\right)$. Putting in the circle $\Rightarrow x^2 + y^2 + 8x - 6y + 9 = 0$

45. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

(A) 8 : 1 (B) 1 : 8 (C) 9 : 1 (D) 1 : 9

Ans : (A)

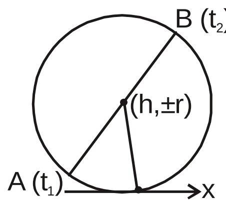
Hint : $1 + \frac{b^2}{a^2} = \frac{9}{8}$ $\Rightarrow \frac{b^2}{a^2} = \frac{1}{8}$ $\Rightarrow \frac{a^2}{b^2} = \frac{8}{1}$

46. Let A, B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Ans : (C,D)

Hint : Slope of AB = $\frac{2}{t_1 + t_2}$



$\therefore \pm r = \frac{2t_1 + 2t_2}{2}$ $\Rightarrow t_1 + t_2 = \pm r$ $\therefore$ Slope = $\frac{2}{r}, \frac{-2}{r}$

47. Let P(at^2, 2at), Q, R(ar^2, 2ar) be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the co-ordinates of K is (2a, 0), then the value of r is

(A) $\frac{t}{1-t^2}$ (B) $\frac{1-t^2}{t}$ (C) $\frac{t^2+1}{t}$ (D) $\frac{t^2-1}{t}$

Ans : (D)

Hint : Slope of QR = $\frac{2}{r - \frac{1}{t}}$

Slope of Pk = $\frac{2at}{at^2 - 2a} = \frac{2t}{t^2 - 2}$ $\Rightarrow \frac{2}{r - \frac{1}{t}} = \frac{2t}{t^2 - 2}$ $\Rightarrow r = \frac{t^2 - 1}{t}$

48. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line through P parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P, Q are on the same side of the x-axis. If R is a point on PQ such that $\frac{PR}{RQ}=\frac{1}{2}$, then the locus of R is

(A) $\frac{x^2}{9}+\frac{9y^2}{49}=1$ (B) $\frac{x^2}{49}+\frac{y^2}{9}=1$ (C) $\frac{x^2}{9}+\frac{y^2}{49}=1$ (D) $\frac{9x^2}{49}+\frac{y^2}{9}=1$

Ans : (A)

Hint : P(3 cosθ, 2sinθ)

Equation of line // to y axis is x = 3 cosθ

It meets circle at θ

$\therefore$ Q(3 cosθ, 3sinθ)

$\because$ PR : RQ = 1 : 2

$\therefore R\left(3\cos\theta, \frac{7\sin\theta}{3}\right)$

$\Rightarrow \frac{x^2}{9}+\frac{9y^2}{49}=1$

49. A point P lies on a line through Q(1, –2, 3) and is parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$. If P lies on the plane 2x + 3y – 4z + 22 = 0, then segment PQ equals to

(A) $\sqrt{42}$ units (B) $\sqrt{32}$ units (C) 4 units (D) 5 units

Ans : (A)

Hint : Let P (+ λ1 ,4 – λ2 ,5λ + 3)

It lies on 2x + 3y – 4z + 22 = 0

$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\therefore 6\lambda = 6$ $\Rightarrow \lambda = 1$

$\therefore$ P = (2, 2, 8) $\therefore PQ=\sqrt{1^2+4^2+5^2}=\sqrt{42}$

50. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, –11, 4) and (2, –3, 1) is

(A) (4, 5, 2) (B) (–4, 5, 2) (C) (4, –5, 2) (D) (4, 5, –2)

Ans : (D)

Hint : Equation of line is

$$\frac{x}{9}=\frac{y+11}{8}=\frac{z-4}{-3}$$

Let foot of ⊥ be P (2λ, 8λ–11, –3λ + 4)

DR's of line joining (1, 8, 4) and 'P' is

(2λ – 1, 8λ – 19, –3λ)

$\therefore 2(2\lambda - 1) + 8(8\lambda - 19) - 3(-3\lambda) = 0$

$\Rightarrow 4\lambda + 64\lambda + 9\lambda = 2 + 152$

$\Rightarrow 77\lambda = 154$ $\Rightarrow \lambda = 2$

$\therefore$ 'P' is (4, 5, –2)

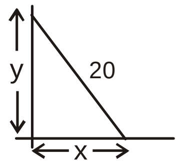
CATEGORY - II (Q51 to Q65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

51. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

(A) $\frac{8}{3}$ (B) $\frac{6}{5}$ (C) $\frac{3}{2}$ (D) $\frac{17}{4}$

Ans : (A)

Hint : 

$x^2 + y^2 = 20^2, \Rightarrow \frac{dx}{dt} = -\frac{y}{x}.\frac{dy}{dt} \Rightarrow \frac{dx}{dt} = -\frac{16}{12}\times(-2) = \frac{8}{3}$

52. For $0 \le p \le 1$ and for any positive a, b ; let $I(p) = (a + b)^p$, $J(p) = a^p + b^p$, then

(A) $I(p) > J(p)$ (B) $I(p) \le J(p)$

(C) $I(p) < J(p)$ in $\left[0, \frac{P}{2}\right]$ & $I(p) > J(p)$ in $\left[\frac{P}{2}, \infty\right)$ (D) $I(p) < J(p)$ in $\left[\frac{P}{2}, \infty\right)$ & $J(p) < I(p)$ in $\left[0, \frac{P}{2}\right]$

Ans : (B)

Hint : let $p = \frac{1}{n}$, then $(a^p + b^p)^{1/p} = \left(a^{\frac{1}{n}} + b^{\frac{1}{n}}\right)^n = a + b + k, k \ge 0 \therefore a^p + b^p \ge (a+b)^p \ge a+b$

53. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}, \vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, whose projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $-\hat{i} - 3\hat{j} - 3\hat{k}$ (B) $\hat{i} - 3\hat{j} - 3\hat{k}$ (C) $-\hat{i} + 3\hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Ans : (A)

54. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three unit vectors such that $\vec{\alpha}.\vec{\beta} = \vec{\alpha}.\vec{\gamma} = 0$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30°. Then $\vec{\alpha}$ is

(A) $2(\vec{\beta} \times \vec{\gamma})$ (B) $-2(\vec{\beta} \times \vec{\gamma})$ (C) $\pm 2(\vec{\beta} \times \vec{\gamma})$ (D) $(\vec{\beta} \times \vec{\gamma})$

Ans : (C)

Hint : $\vec{\alpha} = \lambda(\vec{\beta} \times \vec{\gamma}) \Rightarrow |\vec{\alpha}| = |\lambda(\vec{\beta} \times \vec{\gamma})| \Rightarrow 1 = |\lambda||\beta||\gamma| \sin 30° \Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$

55. Let z_1 and z_2 be complex numbers such that $z_1 \ne z_2$ and $|z_1| = |z_2|$. If Re $(z_1) > 0$ and 1m$(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

(A) One (B) real and positive (C) real and negative (D) purely imaginary

Ans : (D)

Hint : let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2 \therefore |z_1| = |z_2| \Rightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2$ .....(i)

Now $\frac{z_1 + z_2}{z_1 - z_2} = \frac{(x_1^2 - x_2^2) + (y_1^2 - y_2^2) + i.2(x_1y_2 - x_2y_1)}{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ which is purely imaginary (by (i))

56. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

(A) 284×17 (B) 285×17 (C) 284×16 (D) 285×16

Ans : (A)

57. The least positive integer n such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is an identity matrix of order 2 is

(A) 4 (B) 8 (C) 12 (D) 16

Ans : (B)

Hint : $A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, A^8 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\therefore$ correct option is (B)

58. Let ρ be a relation defined on $\mathbb{N}$, the set of natural numbers, as

$\rho = \{(x, y) \in \mathbb{N} \times \mathbb{N} : 2x + y = 41\}$ Then

(A) ρ is an equivalence relation (B) ρ is only reflexive relation

(C) ρ is only symmetric relation (D) ρ is not transitive

Ans : (D)

Hint : $x\rho y$, $y\rho z \Rightarrow 2x + y = 41$ and $2y + z = 41$ which do not imply $2x + z = 41$

59. If the polynomial f (x) = $\begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$, then the constant term of f(x) is

(A) $2 - 3.2^b + 2^{3b}$ (B) $2 + 3.2^b + 2^{3b}$ (C) $2 + 3.2^b - 2^{3b}$ (D) $2 - 3.2^b - 2^{3b}$

Ans : (A)

Hint : $f(0) = \begin{vmatrix} 1 & 2^b & 1 \\ 1 & 1 & 2^b \\ 2^b & 1 & 1 \end{vmatrix} = 2 - 3.2^b + 2^{3b}$

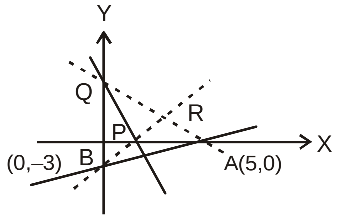
60. A line cuts the x-axis at A (5, 0) and the y-axis at B (0,–3). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P and the y-axis at Q. If AQ and BP meet at R, then the locus of R is

(A) $x^2 + y^2 - 5x + 3y = 0$ (B) $x^2 + y^2 + 5x + 3y = 0$

(C) $x^2 + y^2 + 5x - 3y = 0$ (D) $x^2 + y^2 - 5x - 3y = 0$

Ans : (A)

Hint :



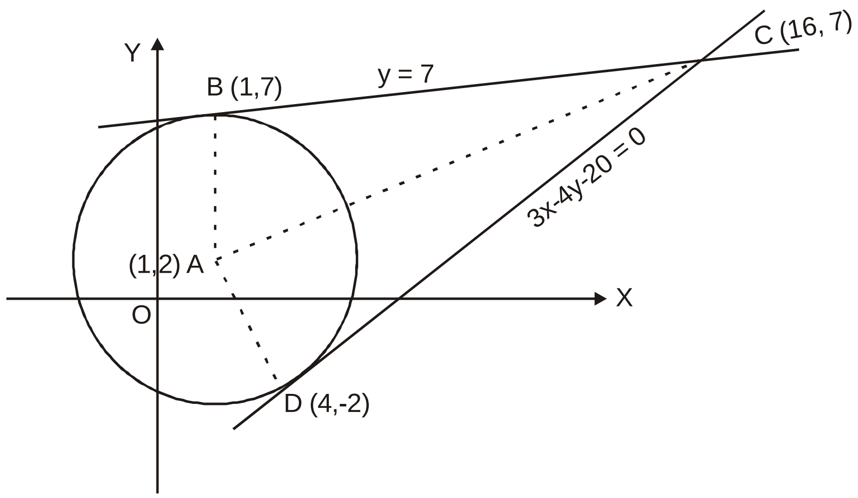
$\angle ARB = \pi/2$

$\therefore$ Locus of R is a circle with A, B as end points of a diameter, which is $x^2 + y^2 - 5x + 3y = 0$

61. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let B (1, 7) and D (4, –2) be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral ABCD is
(A) 150 sq. units (B) 50 sq. units (C) 75 sq. units (D) 70 sq. units

Ans : (C)

Hint :



Area of the quadrilateral ABCD

= 2. ($\triangle$ABC)

= $2.\frac{1}{2}.(15).(5) = 75$

62. Let $f(x) = \begin{cases} -2\sin x, & \text{if } x \le -\frac{\pi}{2} \\ A\sin x + B, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \ge \frac{\pi}{2} \end{cases}$. Then

(A) f is discontinuous for all A and B
(B) f is continuous for all A = –1and B = 1
(C) f is continuous for all A = 1 and B = –1
(D) f is continuous for all real values of A, B

Ans : (B)

Hint : At $x = -\pi/2$, LHL = RHL $\Rightarrow -A + B = 2$

At $x = \pi/2$, LHL = RHL $\Rightarrow A + B = 0$

$\therefore A = -1,\ B = 1 \Rightarrow f(x)$ is continuous $\forall x$

63. The normals to the curve $y = x^2 - x + 1$, drawn at the points with the abscissa $x_1 = 0$, $x_2 = -1$ and $x_3 = \frac{5}{2}$

(A) are parallel to each other (B) are pair wise perpendicular
(C) are concurrent (D) are not concurrent

Ans : (C)

Hint : The three normals are

$x - y + 1 = 0$

$x - 3y + 10 = 0$

$2x + 8y - 43 = 0$

64. The equation x log x = 3 – x
(A) has no root in (1, 3)
(B) has exactly one root in (1, 3)
(C) x log x – (3 – x) > 0 in [1, 3]
(D) x log x – (3 – x) < 0 in [1, 3]

Ans : (B)

Hint : ut

$f(x) = x \log x + x - 3 \qquad f'(x) = \log x + 2 > 0$

$f(1) = -2, \quad f(3) = 3 \log 3, \quad f(1).f(3) < 0$

Hence Exactly one root in $x \in (1,3)$ as $f'(x) > 0$

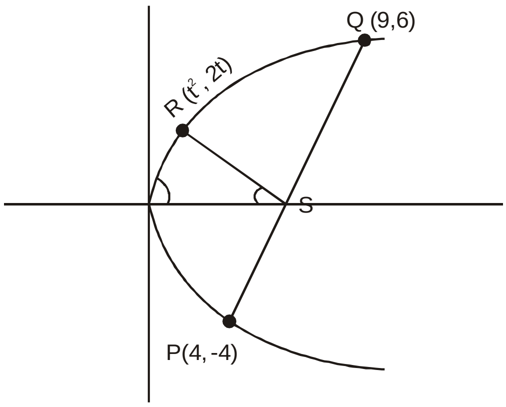
65. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola where P. (4, –4) & Q (9, 6). Let R be a point on the arc of the parabola between P & Q. Then the area of ΔPQR is largest when

(A) $\angle PQR = 90^0$
(B) R (4, 4)
(C) $R\left(\frac{1}{4},1\right)$
(D) $R\left(1,\frac{1}{4}\right)$

Ans : (C)

Hint :

Area is Maximum
When RS is maximum



Equation of PQ is

$y = 2x - 12$

$$RS = \frac{|2t - 2t^2 + 12|}{\sqrt{5}}$$

$$RS = \frac{|2(t^2 - t - 6)|}{\sqrt{5}} = \frac{2}{\sqrt{5}}|(t-3)(t+2)|$$

RS is maximum at $t = 1/2$

Hence R is $(1/4, 1)$

CATEGORY - III (Q66 to Q75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is market then score = 2 x number of correct answers marked $\div$ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will condisered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let I = $\int_0^1 \frac{x^3 \cos 3x}{2+x^2} dx$. Then

(A) $-\frac{1}{2} < I < \frac{1}{2}$ (B) $-\frac{1}{3} < I < \frac{1}{3}$ (C) $-1 < I < 1$ (D) $-\frac{3}{2} < I < \frac{3}{2}$

Ans : (A, B, C, D)

Hint :

67. A particle is in motion along a curve $12y = x^3$. The rate of change of its ordinate exceeds that of abscissa in

(A) $-2 < x < 2$ (B) $x = \pm 2$ (C) $x < -2$ (D) $x > 2$

Ans : (C, D)

Hint : $12\frac{dy}{dt} = 3x^2\frac{dx}{dt}$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \geq 1 \quad \Rightarrow \frac{3x^2}{12} \geq 1, \quad \Rightarrow x^2 \geq 4, \quad \Rightarrow x \in (-\infty, -2] \cup [2, \infty)$

68. The area of the region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ & the parabola $y^2 = ax$, $a > 0$ is

(A) $8\pi a^2$ (B) $a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$ (C) $\frac{16\pi a^2}{9}$ (D) $\pi\left(\frac{27}{8} + 3a^2\right)$

Ans : (B)

Hint : Area $= \frac{\pi a^2}{4} - \frac{2a^2}{3}$

69. If the equation $x^2 - cx + d = 0$ has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

(A) both real (B) both negative

(C) both positive (D) one positive and one negative

Ans : (A, D)

Hint : $x^2 - 4bx + 2b^2 - c = 0$

let α_1, β_1 be roots

$\alpha_1, \beta_1 = 2b^2 - c$

$= 2b^2 - (a^4 - 4a^2b + 2b^2)$

$= a^2(4b - a^2) < 0$

Hence one positive and one negative

70. On the occasion of Dipawali festival each sutdent of a class sends greeting cards to others. If there are 20 students in the class, the number of cards sends by students is

(A) $^{20}C_2$ (B) $^{20}P_2$ (C) $2\times{}^{20}C_2$ (D) $2\times{}^{20}P_2$

Ans : (C, B)

Hint : $^{20}C_2\times 2$

71. In a third order matrix A, a_{ij} denotes the element in the i-th row and j-th column.

If a_{ij} = 0 for i = j

= 1 for i > j

= –1 for i < j

Then the matrix is

(A) skew symmetric (B) symmetric

(C) not invertible (D) non-singular

Ans : (A, C)

Hint : $a_{ij} = -a_{ji}$

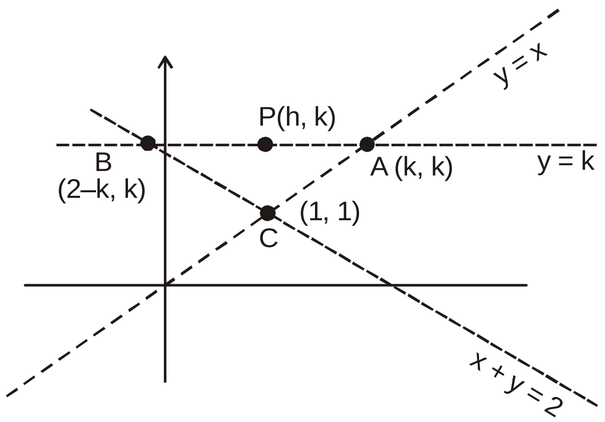
Hence Matrix is skew – symmetric

72. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k), with the lines y = x and x + y = 2 is h^2. The locus of the point P is

(A) x = y – 1 (B) x = –(y – 1) (C) x = 1 + y (D) x = – (1 + y)

Ans : (A, B)

Hint :



Area (Δ ACB) = h^2

$$\Rightarrow \frac{1}{2}\times\sqrt{2(k-1)^2}\times\sqrt{2(k-1)^2}=h^2$$

$$\Rightarrow 4(k-1)^2 = 4h^2$$

$$\Rightarrow y-1=\pm x$$

$$\Rightarrow y-x=1 \text{ and } y+x=1$$

73. A hyperbola, having the transverse axis of length 2 sin θ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

(A) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ (B) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$

(C) $(x^2 + y^2) \sin^2\theta = 1 + y^2$ (D) $x^2 \operatorname{cosec}^2\theta = x^2 + y^2 + \sin^2\theta$

Ans : (B)

Hint : Focus of Ellipse is (1, 0)

For Hyperbola

$a_1 = \sin\theta$

and $a_1 e_1 = 1 \Rightarrow e_1 = \operatorname{cosec}\theta$

$$\Rightarrow b_1^2 = a_1^2\left(e_1^2 - 1\right)$$

$$= \cos^2\theta$$

Equation of Hyperbola is

$$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

74. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$ then assuming k as an integer,

(A) f(x) increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(B) f(x) decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

(C) f(x) decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

(D) f(x) increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

Ans : (A, C)

Hint : $f'(x) = \sin\left(\frac{\pi}{x}\right)\left(\frac{\pi}{x^2}\right)$

$f'(x) > 0 \qquad x \in \left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

$f'(x) < 0 \qquad x \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

75. Consider the function $y = \log_a\left(x + \sqrt{x^2+1}\right)$, $a > 0$, $a \neq 1$. The inverse of the function

(A) does not exist

(B) is $x = \log_{1/a}\left(y + \sqrt{y^2+1}\right)$

(C) is x = sinh (y ln a)

(D) is $x = \cosh\left(-y\ln\frac{1}{a}\right)$

Ans : (C)

Hint : $a^y = x + \sqrt{x^2+1}$

$a^{-y} = \sqrt{x^2+1} - x$

$x = \frac{a^y - a^{-y}}{2} \Rightarrow x = \sinh(y\ln a)$

❑ ❑ ❑